Review Notes - Solving Quadratic Equations

$$ax^2 + bx + c = 0$$

What does solve mean?

Methods for Solving Quadratic Equations:

Solving by using Square Roots

Solving by Factoring using the Zero Product Property

Solving by Quadratic Formula

Solving Quadratic Equations Using Square Roots - Part 1

Definition: If b is a square root of a, then $b^2 = a$.

Example: 4 is a square root of 16 since $4^2 = 16$.

What are the square roots of 16?

Another way to ask the same question is

"What number squared is 16?"

$(4)^2 = 16$ and $(-4)^2 = 16$

There are two answers to this question!!!

16 has two square roots, 4 and -4. 4 is called the positive square root and -4 is the negative square root.

The Radical Symbol: $\sqrt{}$

[The Radical symbol is a grouping symbol.]

$\sqrt{9}$ "The positive square root of 9."

$-\sqrt{9}$ "The negative square root of 9."

$\pm\sqrt{9}$ "The positive and negative square roots of 9."

Every positive number has two(2) square roots.

Every negative number has zero(0) square roots.

Zero has one(1) square root.

Simplify each expression.

Ex 1: $\sqrt{25}$ Ex 2: $-\sqrt{64}$ Ex 3: $3\sqrt{49}$

Ex 4: $-\sqrt{100}+\sqrt{16}$

Ex 5: $6\pm\sqrt{121}$

Ex 6: $-3\pm 2\sqrt{9}$

Ex 7: $\dfrac{-1\pm\sqrt{25}}{2}$

Ex 8: $\dfrac{-3\pm\sqrt{3^2-4(1)(2)}}{2(1)}$

Ex 9: $\dfrac{7\pm\sqrt{(-7)^2-4(6)(-5)}}{2(6)}$

Ex 10: $$\frac{5 \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$$

Solving Quadratic Equations Using Square Roots - Part 2

Simplest Radical Form - SRF

An expression is in Simplest Radical Form if it meets the following conditions.

1. No radicands have perfect square factors other than one. [inside the radical symbol]
2. No radicands contain fractions.
3. No radicals in the denominator.

We use the following properties to simplify radicals.

1. Product Property

2. Quotient Property

3. Multiplicative Identity

Write each expression in SRF and approximate to the nearest hundredth.

Ex 1: $\sqrt{20}$

Ex 2: $\sqrt{72}$

Ex 3: $-\sqrt{300}$

Ex 4: $\sqrt{\dfrac{12}{25}}$

Ex 5: $\sqrt{\dfrac{60}{49}}$

Ex 6: $\sqrt{\dfrac{4}{3}}$

Ex 7: $\sqrt{\dfrac{27}{8}}$

Ex 8: $\sqrt{12} \cdot \sqrt{3}$

Ex 9: $\sqrt{40} \cdot \sqrt{15}$

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Ex 10: $\left(3\sqrt{11}\right)^2$

Ex 11: $\dfrac{4 \pm \sqrt{40}}{2}$

Ex 12: $\dfrac{-6 \pm \sqrt{27}}{3}$

Ex 13: $\dfrac{-8 \pm \sqrt{(8)^2 - 4(1)(-2)}}{2(-2)}$

Solving Quadratic Equations Using Square Roots - Part 3

$0 = ax^2 + bx + c$ can be solved using square roots if b = 0.

This means. $0 = ax^2 + c$

What does solve mean?

Solving Quadratic Equations Using Square Roots

$0 = ax^2 + bx + c$ can be solved using square roots if b = 0.

This means $0 = ax^2 + c$.

Write solutions in SRF. [Isolate x^2.]

Ex 1: $x^2 - 4 = 0$

Ex 2: $x^2 - 64 = 0$

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Ex 3: $x^2 - 16 = 20$

Ex 4: $x^2 + 9 = 59$

Ex 5: $x^2 + 19 = 10$

Ex 6: $3x^2 - 8 = 13$

Ex 7: $12 - 5x^2 = -28$

Ex 8: $-15 + 3x^2 = 5$

Simplify each radical expression. Circle your final answer.

1. $\sqrt{80}$

2. $-\sqrt{125}$

3. $\pm\sqrt{120}$

Simplify each radical expression.
Circle your final answer.

4. $\sqrt{\dfrac{5}{4}}$

5. $\sqrt{\dfrac{2}{27}}$

6. Solve and write your solution in SRF.
$4x^2 - 15 = 60$

How can we factor polynomials?

Factoring refers to writing something as a product. Factoring completely means that all of the factors are relatively prime (they have a GCF of 1).

Methods of factoring:

1. Greatest Common Factor (GCF) - Any polynomial
2. Grouping - Only for 4 or 6 term polynomials
3. Trinomial Method - Only for trinomials
4. Speed Factoring - Special cases only

Method 1: Factoring Out the Greatest Common Factor (GCF)

Factoring out the GCF can be done by using the distributive property.

Ex 1: Factor $12x^3 + 3x^2$.

Step 1: Find the GCF of $12x^3$ and $3x^2$.

The GCF is $3x^2$.

Step 2: Rewrite by factoring out the GCF.

$$3x^2(4x+1)$$

Method 2: Factoring by Grouping

Ex 1: $12xy + 20x + 9y + 15$

Step 1: Group terms together that have a common monomial factor.

Step 2: Factor out the GCF of each group.

Step 3: Find the common polynomial factor and factor it out using the distributive property.

Ex 2: $6xy + 8x - 21y - 28$

Ex 3: $4x^2z^2 - 10x^2 - 6yz + 8yz^2 - 3x^2z - 20y$

Method 3: Factoring Using the Trinomial Method

Step 1: Write the trinomial in descending order.

Step 2: Find two numbers whose product is the same as the product of the first and third coefficients and whose sum is equal to the middle coefficient. (Make a chart.)

Step 3: Rewrite the middle term as the sum of two terms.

Step 4: Use the distributive property and factor by grouping.

Ex 1: $2x^2 - 5x - 3$

Ex 2: $20y^2 + 13yz + 2z^2$

Method 4: Speed Factoring - Special Cases

I. The Difference of Squares

II. Trinomials with a lead coefficient of 1

Special Case: The Difference of Squares

Consider the product: $(a+b)(a-b)$

Ex 1: $x^2 - 121$

Ex 2: $25x^2 - 1$

Ex 3: $72y^2 - 50$

Ex 4: $m^4 - 16$

Special Case: Trinomials with a lead coefficient of 1

$$x^2 + bx + c$$

Find the two numbers whose product is c and whose sum is b.
These are the two numbers in the binomials.

Ex 1: $x^2 + 2x + 1$

Ex 2: $x^2 - x - 12$

Ex 3: $x^2 - 10x + 16$

Ex 4: $x^2 + 28x + 160$

Solving Equations by Factoring - Using the Zero Product Property

The Zero Product Property:

If xy = 0, then either x = 0 or y = 0.

Use the zero product property to solve the following equations.

Ex 1: $x(x-1)=0$

Ex 2: $(x-5)(x+2)=0$

Ex 3: $5x(x-4)=0$

If the polynomial is not "set equal to zero", get all of the terms on one side of the equation first. Then factor the polynomial before trying to use the zero product property to solve.

Ex 4: $x^2 - 3x = 10$

Ex 5: $18 - 3x = x^2$

Ex 6: $w^3 - w^2 = 4w - 4$

Ex 7: $m^3 = 121m$

Factoring Special Products

Old: Difference of Squares

$$a^2 - b^2 = (a-b)(a+b)$$

New: Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

Factor each expression completely.

Ex 1: $h^2 + 4h + 4$

Ex 2: $n^2 - 12n + 36$

Ex 3: $2y^2 - 20y + 50$

Ex 4: $3x^2 + 6xy + 3y^2$

Solve each equation.

Ex 5: $a^2 + 6a + 9 = 0$

Ex 6: $w^2 - 14w + 49 = 0$

Ex 7: $x^2 + \frac{2}{3}x + \frac{1}{9} = 0$

Solving Quadratic Equations by Completing the Square

What can be added to each polynomial so that the expression becomes a square of a binomial?

$x^2 + 8x + \square$	$x^2 - 12x + \square$	$x^2 + 3x + \square$

Solve each quadratic equation by completing the square.

Ex 1: $x^2 - 2x = 3$

Ex 2: $m^2 + 10m = -8$

Ex 3: $3h^2 - 24h + 27 = 0$

Ex 4: $3x^2 - 8x - 10 = 0$

Ex 5: $ax^2 + bx + c = 0$

Solving Quadratic Equations Using the Quadratic Formula

For any quadratic equation $0 = ax^2 + bx + c$,

the solution(s) are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Step 1: Write the equation in standard form.
(Set equal to zero and in descending order)

Step 2: Identify all coefficients. a = ____, b = ____, c = ____.

Step 3: Substitute a, b, and c into the formula and simplify.

Solve each equation using the quadratic formula.
Circle your final answer and use SRF if necessary.

Ex 1: $x^2 + 8x - 1 = 0$

Ex 2: $x^2 + 6x = 5$

Ex 3: $3n^2 = 5n - 1$

Ex 4: $5x^2 + 12x + 10 = 9 + 9x$